On the Skyrme model prediction for the N-N spin-orbit force

Abdellatif Abada*

Theoretical Physics Group, Department of Physics and Astronomy

University of Manchester, Manchester M13 9PL, UK

Abstract

In the framework of the product ansatz as an approximation for the two-baryon system we review in details the derivation of the isoscalar nucleon-nucleon spin orbit potential coming from the sixth order term of the extended Skyrme model. We show that the sixth order term contributes with a positive sign, as is the case for the Skyrme term, contrary to the claims of Riska and Schwesinger. Those authors considered only one part of the force due to the sixth order term and omitted the second part which turns out to be the dominant one. Our result is independent of the parameters of the model.

PACS: 11.10Lm, 12.39Dc, 13.75Cs.

Keywords: Effective Lagrangian, Skyrmion, N-N spin-orbit force, Baryon current.

MC/TH 96/01 January 1996

*e-mail: abada@a13.ph.man.ac.uk

There have been several attempts [1, 2] to extract the N-N spin-orbit interaction from the standard Skyrme Lagrangian [3] which includes the non-linear σ model supplemented with a stabilizing fourth-order term in powers of the derivatives of the pion field. The calculations were based on the product ansatz for the two-baryon system as suggested by Skyrme [4]. The advantage of this approximation is that, beyond its relative simplicity as compared to other two-baryon field configurations which can be found in the literature, it becomes exact for large N-N separation[1]. Therefore one can expect the asymptotic behaviour of the N-N spin-orbit force derived from the Skyrme model by using the product ansatz agrees with the phenomenology. Unfortunately this is not the case for the isoscalar component of the spin-orbit force. All the authors who worked on that subject agree on the result that the standard Skyrme model predicts an isospin independent spin-orbit force with the *wrong* sign. Namely, it predicts a *repulsive* interaction while the phenomenological Bonn potential [5] as well as the Paris potential [6] give an *attractive* one. A natural reaction that one might have in order to cure this illness, is to improve the Skyrme model. Indeed, it has recently been shown [7, 8] through the study of some properties of the nucleon, that in order to describe properly low-energy hadron physics one should not restrict oneself to the standard Skyrme model but consider extensions of this model including higher order terms in powers of the derivatives of the pion field. Expressed in terms of an SU(2) matrix U which characterizes the pion field, a 6th-order term corresponding to ω-meson exchange [9],

$$\mathcal{L}_6 = -\frac{\beta_\omega^2}{2m_\omega^2} B_\mu(U) B^\mu(U) \ , \tag{1}$$

where $B^\mu = \epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\,(\ (\partial_\nu U)U^+(\partial_\alpha U)U^+(\partial_\beta U)U^+\ )/24\pi^2$ is the baryon current [3], m_ω the ω-meson mass and β_ω a dimensionless parameter related to the $\omega \to \pi\gamma$ width, might be a good candidate to solve the problem of the N-N isoscalar spin-orbit force. Riska and Schwesinger [10] at first and Kälbermann and Eisenberg [11] more recently examined the influence of such term on the spin orbit interaction[2]. These authors claimed that the inclusion of the sixth-order term leads to the correct sign (attractive interaction) for the isoscalar spin-orbit potential. However they considered only one part of the interaction due to the sixth-order term in their calculations and omitted the second part which arises from the exchange current [10, 12]. The aim of this

[1]The region of validity of the product ansatz corresponds to a relative distance r larger than 1 fm

[2]A dilaton field has also been included in [11].

paper is to display the derivation of the isoscalar spin-orbit potential and show that the term omitted in [10] contributes significantly to that force.

For a system of two interacting solitons, Skyrme [4] suggested the use of the product ansatz. Rotational dynamics are also introduced to obtain the appropriate spin and isospin structure [13]. Thus, the field configuration of the two-nucleon system separated by a vector $\mathbf{r}$ can be written

$$\begin{aligned} U(A_1, A_2, \mathbf{x}, \mathbf{r}) = U_1 U_2 = A_1 U_H(\mathbf{r}_1) A_1^{+} A_2 U_H(\mathbf{r}_2) A_2^{+} \quad , \\ \mathbf{r}_1 = \mathbf{x} - \mathbf{r}/2 \quad , \quad \mathbf{r}_2 = \mathbf{x} + \mathbf{r}/2 \quad , \end{aligned} \tag{2}$$

where A_1 and A_2 are SU(2) matrices. To carry out a simultaneous quantization of the relative motion of the two nucleons and the rotational motion we need to treat $\mathbf{r}$, A_1 and A_2 as collective coordinates. Hence we make all these parameters ($\mathbf{r}$, A_1, A_2) time dependent. In Eq. (2) U_H is the commonly used SU(2) matrix for a single soliton with the hedgehog ansatz:

$$U_H(\mathbf{x}) = \exp[\, i\boldsymbol{\tau}.\hat{\mathbf{x}} F(|\mathbf{x}|)\,] \quad , \quad \hat{\mathbf{x}} \equiv \mathbf{x}/|\mathbf{x}| \quad , \tag{3}$$

where $F(|\mathbf{x}|)$ obeys the usual boundary conditions for winding number one, and the τ_a's are the Pauli matrices. Let us observe that the chiral angle F, which will be used below in the numerical calculations, is obtained by solving the static Euler-Lagrange equation derived from the extended Skyrme Lagrangian including the non-linear σ model, the 4th-order term, the 6th-order term as well as a small chiral symmetry breaking term [8].

The spin-orbit potential will emerge due to a coupling between the relative motion and the spins of the two nucleons so that we have to calculate the kinetic energy corresponding to (1). As it is given in Ref. [10] it reads

$$K_6(\mathbf{r}) = -\frac{\beta_\omega^2}{2m_\omega^2} \int \mathrm{d}^3x \; \mathbf{B}^2(U_1 U_2) \quad , \tag{4}$$

where $\mathbf{B}$ is the spatial-component of the baryon current defined after Eq. (1). Before going further, a word of caution should be given here. In principle before identifying and extracting the spin-orbit potential one has to treat carefully the conversion from velocities (orbital and rotational) to canonical momenta (orbital momentum and spin). Namely one has to start from a Lagrangian formalism, consider the "classical" kinetic energy (which is the opposite of Eq. (4) in the case of (1)), calculate the mass matrix and then invert it properly in order to move to a Hamiltonian formalism [2]. However for a large relative distance r, the region of

validity of the product ansatz, this procedure [2] is equivalent to that of Refs. [1, 10] which we will use here [14]. In the latter one starts from Eq. (4), in our case, and make the usual identifications [13, 15]

$$\dot{\mathbf{r}}_n \to \frac{\mathbf{p}^{(n)}}{M} \quad , \quad \boldsymbol{\omega}_n = -\frac{i}{2}\mathrm{Tr}(\boldsymbol{\tau} A_n^+ \dot{A}_n) \to \frac{\mathbf{s}^{(n)}}{2\lambda} \quad , \quad n = 1, 2 \quad , \tag{5}$$

where $\mathbf{p}^{(n)}$ and $\mathbf{s}^{(n)}$ are respectively the radial momentum and the spin of the n-th nucleon while M and λ are the mass and the moment of inertia of the single soliton.

Inserting the product ansatz (2) into the spatial components of the baryon current gives

$$\mathbf{B}(U_1 U_2) = \mathbf{B}(U_1) + \mathbf{B}(U_2) + \mathbf{B}_{\mathrm{ex}}(U_1, U_2) \quad , \tag{6}$$

where $\mathbf{B}(U_n)$ and $\mathbf{B}_{\mathrm{ex}}(U_1, U_2)$ are the single baryon current and the exchange current densities, respectively. The single baryon current is given by

$$\mathbf{B}(U_n) = B_0(|\mathbf{r}_n|)\left(\frac{\mathbf{p}^{(n)}}{M} + \mathbf{r}_n \times \frac{\mathbf{s}^{(n)}}{\lambda}\right) \quad \text{for} \quad n = 1, 2 \quad , \tag{7}$$

where $B_0(r) = -F'(r)\sin^2 F(r)/2\pi^2 r^2$ is the baryon density, while the rather complicate expression of $\mathbf{B}_{\mathrm{ex}}$ reads

$$B_{i\ \mathrm{ex}} = D_{pq}(C)\ \left(\mathcal{A}_{ipql}\ \frac{s_l^{(1)}}{2\lambda} + \mathcal{A}'_{ipql}\ \frac{p_l^{(1)}}{M} + \mathcal{B}_{ipql}\ \frac{s_l^{(2)}}{2\lambda} + \mathcal{B}'_{ipql}\ \frac{p_l^{(2)}}{M}\right) \quad , \tag{8}$$

in which $C = A_1^+ A_2$ and D_{pq} is the 3×3 rotation matrix in the adjoint representation

$$D_{pq}(C) = \frac{1}{2}\mathrm{Tr}(\tau_p C \tau_q C^+) \quad . \tag{9}$$

The sum from 1 to 3 on repeated indices in Eq. (8), and from here on, is understood. The four tensors appearing in the expression of $\mathbf{B}_{\mathrm{ex}}$ depend only on the positions of the two nucleons. They read

$$\begin{aligned}
\mathcal{A}_{ipql}(\mathbf{r}_1, \mathbf{r}_2) = &\ \left(2\epsilon_{ijk}\epsilon_{abp}\ T_{lb}^{(1)}\ R_{ja}^{(1)}\ R_{qk}^{(2)} - T_{lp}^{(1)}\ \alpha_{iq}^{(2)}\right)/4\pi^2 \quad , \\
\mathcal{A}'_{ipql}(\mathbf{r}_1, \mathbf{r}_2) = &\ \left(2\epsilon_{ijk}\epsilon_{abp}\ R_{lb}^{(1)}\ R_{ja}^{(1)}\ R_{qk}^{(2)} - R_{lp}^{(1)}\ \alpha_{iq}^{(2)}\right)/4\pi^2 \quad , \\
\mathcal{B}_{ipql}(\mathbf{r}_1, \mathbf{r}_2) = &\ \left(-2\epsilon_{ijk}\epsilon_{abq}\ T_{bl}^{(2)}\ R_{kp}^{(1)}\ R_{aj}^{(2)} + T_{ql}^{(2)}\ \alpha_{pi}^{(1)}\right)/4\pi^2 = -\mathcal{A}_{iqpl}(-\mathbf{r}_2, -\mathbf{r}_1) \quad , \\
\mathcal{B}'_{ipql}(\mathbf{r}_1, \mathbf{r}_2) = &\ \left(2\epsilon_{ijk}\epsilon_{abq}\ R_{bl}^{(2)}\ R_{kp}^{(1)}\ R_{aj}^{(2)} - R_{ql}^{(2)}\ \alpha_{pi}^{(1)}\right)/4\pi^2 = \mathcal{A}'_{iqpl}(-\mathbf{r}_2, -\mathbf{r}_1) \quad ,
\end{aligned} \tag{10}$$

where

$$
\begin{aligned}
&R^{(n)}_{ia} = -\frac{i}{2}\mathrm{Tr}(\tau_a U^+_H(\mathbf{r}_n)\partial_i U_H(\mathbf{r}_n)) = r_n\left(c_n s_n(\delta_{ia}-\hat{\mathbf{r}}_{ni}\hat{\mathbf{r}}_{na}) + \frac{F'_n}{r_n}\hat{\mathbf{r}}_{ni}\hat{\mathbf{r}}_{na} + s^2_n\epsilon_{ial}\hat{\mathbf{r}}_{nl}\right) , \\
&T^{(n)}_{bc} = 2\ \epsilon_{ilb}\ r_{nl}\ R^{(n)}_{ic} = 2\ r^2_n\left(-s^2_n(\delta_{bc}-\hat{\mathbf{r}}_{nb}\hat{\mathbf{r}}_{nc}) + s_n c_n\ \epsilon_{bcl}\ \hat{\mathbf{r}}_{nl}\right) , \\
&\alpha^{(n)}_{ia} = \epsilon_{ijk}\ \epsilon_{abc}\ R^{(n)}_{bj}\ R^{(n)}_{ck}\ , \\
&r_n \equiv |\mathbf{r}_n|\ ,\quad F_n \equiv F(r_n)\ ,\quad F'_n \equiv \frac{\partial F_n}{\partial r_n}\ ,\quad c_n \equiv \frac{\cos F_n}{r_n}\ ,\quad s_n \equiv \frac{\sin F_n}{r_n}\ ,\qquad n = 1,2\ .
\end{aligned}
\tag{11}
$$

By inserting now the expression (6) in the kinetic energy (4) we obtain

$$
K_6(\mathbf{r}) = -\frac{\beta^2_\omega}{2m^2_\omega}\int \mathrm{d}^3x\ \left(2\mathbf{B}(U_1).\mathbf{B}(U_2) + \mathbf{B}^2_{\text{ex}} + 2\ (\mathbf{B}(U_1)+\mathbf{B}(U_2)\ )\ .\mathbf{B}_{\text{ex}}\ +\ \cdots\right)\ , \tag{12}
$$

where we omitted the terms corresponding to the sum of the squares of the single baryon currents since they do not bring any coupling between the two nucleons and thus do not contribute to the spin-orbit force. From Eqs. (8) and (9) one sees that the exchange current $\mathbf{B}_{\text{ex}}$ contains the isospin factor $\boldsymbol{\tau}^{(1)}.\boldsymbol{\tau}^{(2)}$ due to the projection theorem [16]

$$
\langle N'_1 N'_2|D_{ab}(C)|N_1 N_2\rangle = \frac{1}{9}(\boldsymbol{\tau}^{(1)}.\boldsymbol{\tau}^{(2)})\sigma^{(1)}_a\sigma^{(2)}_b\ , \tag{13}
$$

while the single baryon currents are isospin independent [cf., Eq. (7)]. Therefore the isoscalar component of the spin-orbit force arises only from the first and the second term in the expression (12). The calculation of the spin-orbit potential derived from the first term is straightforward. By inserting the definition of the single currents (7) and keeping the terms proportional to $\mathbf{L}.\mathbf{S}$ where $\mathbf{S} = \mathbf{s}^{(1)} + \mathbf{s}^{(2)}$ is the total spin and $\mathbf{L} = \mathbf{r}\times\mathbf{p}$, is the angular momentum, $\mathbf{p}$ being the relative momentum, i.e., $\mathbf{p} = \mathbf{p}^{(2)} = -\mathbf{p}^{(1)}$, we obtain

$$
-\frac{\beta^2_\omega}{2m^2_\omega}\int \mathrm{d}^3x\ 2\mathbf{B}(U_1).\mathbf{B}(U_2)\ \to \frac{\beta^2_\omega}{2m^2_\omega}\frac{1}{M\lambda}\Sigma_6(r)\ \mathbf{L}.\mathbf{S}\ , \tag{14}
$$

where

$$
\Sigma_6(r) = -\int \mathrm{d}^3x\ B_0(r_1)B_0(r_2)\ = -\frac{1}{4\pi^4}\int \mathrm{d}^3x\ s^2_1 s^2_2 F'_1 F'_2\ . \tag{15}
$$

The above formulae are equivalent to that obtained in Ref. [10] and the function Σ_6 as defined in Eq. (15) is obviously negative (see Fig.1).

The second term in the kinetic energy (12) has been omitted in the calculations of Ref. [10]. In fact in their paper Riska and Schwesinger [10] referred to the article [12] for the expression of the exchange baryon current. However in that paper [12] the momentum dependent terms,

as given here in Eq. (8), have been dropped from the exchange current, since their contribution is of less significance than the spin dependent terms for the calculation of the deuteron form factors, which was the purpose of the article [12]; and thus no spin-orbit coupling will arise from their incomplete exchange current [10]. This is obviously not the case when one takes into consideration the entire expression (8). Even though the exchange current (8) is proportional to the rotation matrix D_{pq}, and thus to $\boldsymbol{\tau}^{(1)}.\boldsymbol{\tau}^{(2)}$, $\mathbf{B}^2_{\rm ex}$ contains an isoscalar component. Indeed one has the formula

$$D_{pq}(C)D_{p'q'}(C) = \frac{1}{3}\delta_{pp'}\delta_{qq'} + \frac{1}{2}\epsilon_{pp'p''}\ \epsilon_{qq'q''}\ D_{p''q''}(C) + \sum_{mn} C^{mn}_{pqp'q'}\mathcal{D}^{j=2}_{mn}(C) \quad , \tag{16}$$

which can be obtained by expressing the $D_{pq}(C)$ in terms of the Wigner $\mathcal{D}$-functions. By replacing now $D_{pq}(C)D_{p'q'}(C)$ by $\frac{1}{3}\delta_{pp'}\delta_{qq'}$ (we are indeed interested only in the isoscalar part of the spin-orbit force) in the expression of $\mathbf{B}^2_{\rm ex}$ and keeping only the relevant terms which might generate an $\mathbf{L}.\mathbf{S}$ contribution we obtain

$$-\frac{\beta^2_\omega}{2m^2_\omega}\int {\rm d}^3x\ \mathbf{B}^2_{\rm ex} \ \rightarrow\ \frac{\beta^2_\omega}{2m^2_\omega}\frac{1}{M\lambda}\ S_l\left(-\frac{1}{3}\int {\rm d}^3x\ \mathcal{A}_{ipql}(\mathcal{B}'_{ipql'} - \mathcal{A}'_{ipql'})\right)p_{l'} \quad . \tag{17}$$

After some tedious calculations[3] [cf. Eqs. (10,11)], the term between brackets in the above equation turns out to have the structure $\Sigma_{6\rm ex}(r)\ \epsilon_{il'l}\ r_i$ so that one obtains

$$-\frac{\beta^2_\omega}{2m^2_\omega}\int {\rm d}^3x\ \mathbf{B}^2_{\rm ex} \ \rightarrow\ \frac{\beta^2_\omega}{2m^2_\omega}\frac{1}{M\lambda}\ \Sigma_{6\rm ex}(r)\ \mathbf{L}.\mathbf{S} \quad . \tag{18}$$

The expression of $\Sigma_{6\rm ex}$ reads

$$\Sigma_{6\rm ex} = \frac{1}{3\pi^4 r}\int {\rm d}^3x\ r_1 s_1^2\Big(\frac{1}{4}\hat{\mathbf{r}}.\hat{\mathbf{r}}_1\ (\hat{\mathbf{r}}_1.\hat{\mathbf{r}}_2)^2(s_1^2 - F_1'^2)(s_2^2 - F_2'^2) + \frac{1}{4}\hat{\mathbf{r}}.\hat{\mathbf{r}}_2\ \hat{\mathbf{r}}_1.\hat{\mathbf{r}}_2(F_2'^2 - s_2^2)\times$$
$$(2F_1'^2 + s_1^2 + s_2^2)\ + \hat{\mathbf{r}}.\hat{\mathbf{r}}_1\ (\ \frac{3}{2}s_2^2F_1'F_2' - \frac{3}{4}F_1'^2F_2'^2 - \frac{1}{2}s_1^2F_2'^2 - \frac{3}{4}s_2^2F_1'^2 - \frac{7}{4}s_2^2F_2'^2 - \frac{3}{4}s_2^4 - s_1^2s_2^2)\Big) \tag{19}$$

We refer to Eqs. (3,11) for the notations used in the above formula. Thus the isospin independent spin-orbit force generated by the 6th-order term in powers of the derivatives of the pion field (1) reads

$$V_{\rm SO} \equiv \frac{\beta^2_\omega}{2m^2_\omega}\frac{1}{M\lambda}\ \Sigma_{6\rm tot}(r)\ \mathbf{L}.\mathbf{S} = \frac{\beta^2_\omega}{2m^2_\omega}\frac{1}{M\lambda}\ \left(\Sigma_6(r) + \Sigma_{6\rm ex}(r)\right)\ \mathbf{L}.\mathbf{S} \quad , \tag{20}$$

[3] These have been checked by computer algebra.

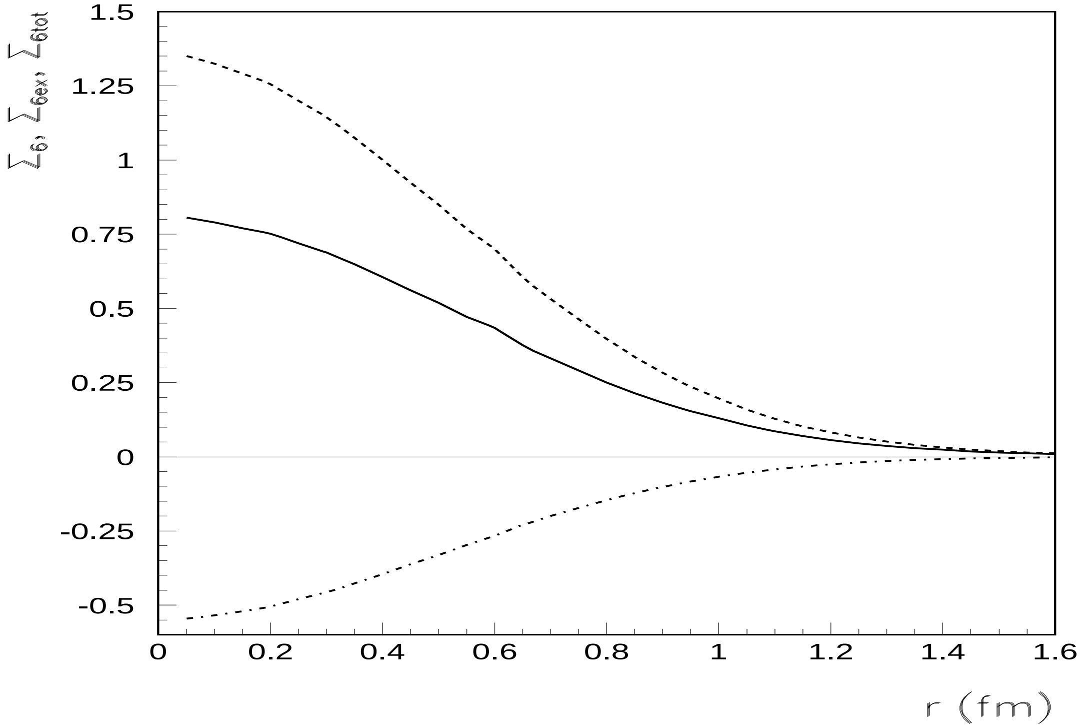

FIG. 1 : The functions Σ_6 (dashed-dotted line), $\Sigma_{6\rm ex}$ (dashed line) and $\Sigma_{6\rm tot}$ (full line) in fm^{-3}, as given in Eqs. (15,19, 20), with respect to the relative N-N distance r.

where the expressions of $\Sigma_6(r)$ and $\Sigma_{6\rm ex}(r)$ are given in Eqs. (15) and (19) respectively. In Fig.1 we plot these two functions, with respect to the relative distance r. We see from that figure that $\Sigma_6(r)$ is negative, as it was expected and found in [10]. However the contribution of the exchange current which has been omitted in [10], $\Sigma_{6\rm ex}$, is positive and larger than $\Sigma_6(r)$ so that the total function $\Sigma_{6\rm tot}$, also displayed in Fig.1, is always positive. Therefore the 6th-order term generated by ω-meson exchange (1) gives a *repulsive* isoscalar spin-orbit interaction, contrary to the claims of Ref. [10]. Our result is independent of the parameters β_ω, m_ω as one can see from Eq. (20). Of course the chiral angle F on which the functions Σ depend, depends implicitly on the parameters of the extended Skyrme model [3, 8] (f_π, e, β_ω). Nevertheless the main result of this paper, namely, the dominance of the positive contribution $\Sigma_{6\rm ex}$ over the negative one Σ_6, is independent of the parameters of the model. We checked this result numerically by considering several sets of parameters which can be found in the literature; realistic as well as unrealistic ones.

In this letter we derived in details the isospin-independent spin-orbit force from the sixth-order order term in powers of the derivatives of the pion field (1). By taking into account the results of Refs. [1, 2] concerning the Skyrme term [3], we arrive to our major conclusion: neither the Skyrme term which can be derived from a local approximation of an effective ρ model, nor the sixth-order term generated by ω-meson exchange, reproduces the phenomenological isoscalar N-N spin-orbit interaction. Indeed both terms give a *repulsive* force while it should be *attractive* according to [5, 6]. Hence the problem of the sign for this force is still unsolved and one should try to understand why. The key to the problem is not at hand yet but one can suggest some hints. The spin-orbit force is a relativistic problem and since we are in a region of large distance (the region of validity of the product ansatz) where this force is weak, any relativistic effects, even small ones, can affect that interaction. Then one should consider such relativistic effects in the calculations (we are thinking, e.g., to the Thomas precession effects due to the rotation of the nucleons). Also one sees from the Bonn potential [5] that the scalar degrees of freedom, namely the σ-meson which can be viewed as the responsible for the enhancement of the $\pi\pi$ S-wave, provide one of the major contributions to the isoscalar spin-orbit force. So maybe one has to investigate the spin-orbit part of the two-pion exchange potential within the Skyrme model [17] in order to correct the anomaly of that sign. This idea to account for a scalar field has been investigated in Ref. [11] in which a dilaton field is coupled to Skyrmions in order to mimic the scale breaking of QCD. Even though it has been pointed out in Ref. [18] that a dilaton field is not suitable to provide a good description of low-energy hadron physics, the fact remains that a combination of the sixth-order term (1) and dilaton coupling, i.e., *scalar* meson coupling, yields an attractive isoscalar spin-orbit force as it has been claimed in [11]. However this result remains questionable since these authors, as those of Ref. [10], did not take into account the exchange current contribution.

Acknowledgments

We express our deep appreciation to M. C. Birse for helpful discussions. We also thank him as well as J. McGovern for a critical reading of the manuscript.

References

[1] E. M. Nyman and D. O. Riska, Phys. Lett. B **175** (1986) 392 ; **183** (1987) 7 ; D. O. Riska and K. Dannbom, Phys. Scr. **37** (1988) 7 ; T. Otofuji *et al*, Phys. Lett. B **205** (1988) 145.

[2] R. D. Amado, B. Shao and N. R. Walet, Phys. Lett. B **314** (1993) 159 and an erratum : Phys. Lett. B **324** (1994) 467 ;
B. Shao, N. R. Walet and R. D. Amado, Phys. Rev. C **48** (1993) 2498 and an erratum : Phys. Rev. C **49** (1994) 3360.

[3] T. H. R. Skyrme, Proc. Roy. Soc. **A260** (1961) 127.

[4] T. H. R. Skyrme, Nucl. Phys. **31** (1962) 556.

[5] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. **149** (1987) 1.

[6] M. Lacombe *et al*, Phys. Rev. C**21** (1980) 861.

[7] B. Moussallam, Ann. Phys. (N.Y.) **225** (1993) 264.

[8] A. Abada and H. Merabet, Phys. Rev. D **48** (1993) 2337.

[9] A. Jackson *et al*, Phys. Lett. B **154** (1985) 101.

[10] D. O. Riska and B. Schwesinger, Phys. Lett. B **229** (1989) 339.

[11] G. Kälbermann and J. M. Eisenberg, Phys. Lett. B **349** (1995) 416.

[12] E. M. Nyman and D. O. Riska, Nucl. Phys. A **468** (1987) 473.

[13] G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. **B228** (1983) 552.

[14] D. O. Riska, private correspondence.

[15] E. M. Nyman, Phys. Lett. B **162** (1985) 244.

[16] T. Walhout and J. Wambach, Int. Jou. Mod. Phys. E Vol. 1, No. 4 (1992) 665.

[17] R. Vinh Mau, private communication; N. R. Walet, private communication.

[18] M. C. Birse, J. Phys. G **20** (1994) 1287.